Next Generation Spectrum Regulation: Price-Guided Radio Policy

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Abstract: This article explains how market signals in the form of pricing information can be introduced into spectrum management in order to optimally guide not only assignment, but also determinations concerning type of use, emissions characteristics and exclusivity. It presents a mathematical model to illuminate how one possible implementation of such price-guided policy might function to make these determinations. As compared to conventional spectrum auctions, price-guided mechanisms for determining allocation and policy would arrive at an assignment of spectrum rights to the highest value users as well as ensure that the contours of those rights were the most efficient possible. In the mathematical model, participants in a hypothetical auction are free to express their demand for spectrum licences which are different on several dimensions such as permissible power output and bandwidth. The most actionable initial implementations of this new approach include determinations of maximum power limits, bandwidth, duration of rights and channelisation. Other early potential implementations include boundary interference standards and possibly congestion-based protocols. Price-guided policy holds substantial promise because it encourages allocative efficiency of spectrum due to the fact that bidders can acquire exactly the set of spectrum rights they need. Further, price-guided policy mitigates the allocation errors inherent in administrative determinations.

Key words: spectrum policy, auctions, price-guided policy, regulation.

For centuries, auctions have been used to assign objects to their highest financial value user. However, even successful auctions typically generate only two new pieces of information: (1) the valuation of the object and (2) by inference, to whom the object should be assigned. This article examines how an auction could be used to generate other pieces of information such as certain characteristics of the object to be assigned through an auction. In particular, it models how market signals in the form of pricing information can be introduced into radio spectrum management in order to optimally guide not only assignment, but also determinations concerning type of use, emissions and modulation characteristics, and exclusivity.

The primary social objective of spectrum policy is to maximize the benefit which society obtains from use of the limited radio spectrum. And yet, for most of the Twentieth Century, spectrum policy tightly controlled all aspects of radio operation. It did so by imposing strict conditions on the rights of use. These restrictions have had the unintended effect of limiting market entry and flexibility of use; therefore, limiting the potential benefit of radio operations.

Since radio operations are a critical input into all areas of private economic and public activity, governments around the world have been engaged in a radical rethinking of spectrum policy, moving away from the stodgy, decades-old, regulatory regime towards approaches which afford greater flexibility. The more flexible regimes envisioned are based on technological and market-oriented solutions to achieve a more optimal level of allocation to particular services. This move towards technical and economic flexibility has also been fuelled by advances in radio signal processing afforded by ever increasing computing power. The idea to use market mechanisms to determine spectrum assignment is not new. It began to take shape in 1959, when the Federal Communications Commission called Ronald Coase to testify about his proposal for market assignment of radio spectrum rights (COASE, 1959).

Spectrum management authorities have had to address the appropriate means of exclusion of rival uses for spectrum because it is impossible to exclude or limit the use of a common resource such as spectrum. Without exclusion, users consume the spectrum without regard to the fact that their usage causes the deleterious effect of interference for other would-be users. They, therefore, tend to overuse the spectrum, reducing the benefits obtained by all, an outcome referred to as the Tragedy of the Commons (HARDIN, 1968). Historically, spectrum management authorities have used administrative proceedings to select uses and users. Through this selection process, spectrum management authorities can coordinate behavior to reduce the likelihood that spectrum will become over used (MARCUS *et al.*, 2006).

The determination of users is principally achieved through the creation and assignment of rights to emit radio frequency energy *and independently* the right to be free from the interfering radio energy of other users. As an implicit part of this inquiry, spectrum management authorities must set certain operational parameters for spectrum use. These parameters establish permissible emissions power, operating frequencies, and guard bands, *inter alia*.

Spectrum management authorities are making many of these decisions as to use and users purely as administrative determinations, in the absence of information about the monetary valuation of the possible alternatives. [1] Price-guided policy is a means of making policy determinations whereby administrative decisions are supplemented with pricing or market information, usually in the form of auctions.

In this article, I present a mathematical model to demonstrate how market information in the form of price signals can be used to establish efficient parameters for spectrum regulations beyond just the assignment of rights of use. While not supplanting the decisions of the regulator, price discovery mechanisms such as auctions are an effective tool for rationalising administrative determinations and could establish usage parameters which are more economically efficient than those established by administrative proceeding alone.

Spectrum regulatory processes

Figure 1 shows the process of spectrum policy development, moving right as it matures. [2]

Spectrum policy begins with allocation. Here the spectrum management authority identifies, usually through an audit, bands available for use. At this stage, decisions concerning use, such as whether the band will be used for mobile, nomadic (portable) or fixed, are made. This has a profound impact on the network architecture of the future service. In the policy phase, the spectrum management authority makes the rules which will govern operation in the band. These decisions include the establishment of rules defining modulation characteristics, bandwidths, channelisation of blocks within a band, power limits, permissible interference, tower siting rules, licence duration and RF safety. Allocation and policy determinations affect:

- the spectrum band which can be used;
- the geographical area where the spectrum band can be used; and
- the period of time when spectrum can be used.

[1] The inherent issues in administrative determinations are discussed later.

[2] Compare, POGOREL, 2007, at p. 171 (providing different definitions of the spectrum policy process).

Heretofore, the allocation and policy stages were accomplished by administrative rule-makings.

Figure 1 - Spectrum regulatory processes

Process	Allocation	Policy	Assignment	Oversight
Decisions/Actions	•Band Selection •Use decisions •Network architecture	•Modulation and emissions limits •Power limits •Interference •Tower siting •RF Safety	•Identification of Users •Awarding of rights through licenses	•Police Role
Mechanisms	Administrative Rulemaking	Administrative Rulemaking	Auctions Lotteries Comparative Hearings	Complaint Resolution Disciplinary/ Actions
Price-Guided	Auctions Spectrum Trading	Auctions	Auctions Spectrum Trading	Complaint Resolution Disciplinary/ Actions

Source: WIK-Consult

In the assignment phase, those who are granted usage rights are identified, and permissions are granted. These are normally individual rights and conveyed in the form of a licence. At present, price-guided policy in the form of auctions is used to determine assignment by numerous spectrum management authorities around the world. Administrative tools such as comparative hearings and lotteries have been widely used to assign such rights. Pioneer preferences – first in time, first in right – have also been used. Usage rights are typically assigned as exclusive rights. However, general authorisations, which grant rights by licence to a limited number of individuals or to all comers, are also possible. Examples of licences in a shared regime include business licences, drivers' licences, or Ham Radio operator's licences. [3] The oversight stage represents the spectrum

[3] It is often wrongly assumed that shared and licensed approaches to spectrum management are collective exhaustive and mutually exclusive (SNIDER, 2006). Given that these approaches are not mutually exclusive, there has to date been very little work done on finding an optimal

management authority's police role. These actions are accomplished by complaint resolutions, disciplinary actions and, at times, legal proceedings. It would present perverse public policy outcomes to introduce market forces into the oversight role since it enables financial incentives to influence adjudications. This behavior is effectively bribery.

Band plan determinations

The spectrum management authority must make band plans for various radio spectra. These determinations include setting rules for frequencies such as the size of spectrum blocks within a band, maximum power limits, placing and width of guard bands, and pairing decisions. Since all usable spectrum has been allocated and assigned, this process normally begins with an audit for underutilized spectrum resources. Once a potentially free spectrum band is identified, the spectrum management authority must decide certain parameters of the radio use within this band. These parameters include the maximum permissible power, the width of spectrum blocks to be allocated and assigned, the spacing of those spectrum blocks, the width and spacing of guard bands, and the pairing or lack of pairing of assigned blocks. Determinations of modulation characteristics can also be used by spectrum management authorities and can control the amount of RF energy emitted which could result in harmful interference. These determinations must be made *ex ante* and are closely tied to considerations such as network architecture, radio technology, and usage.

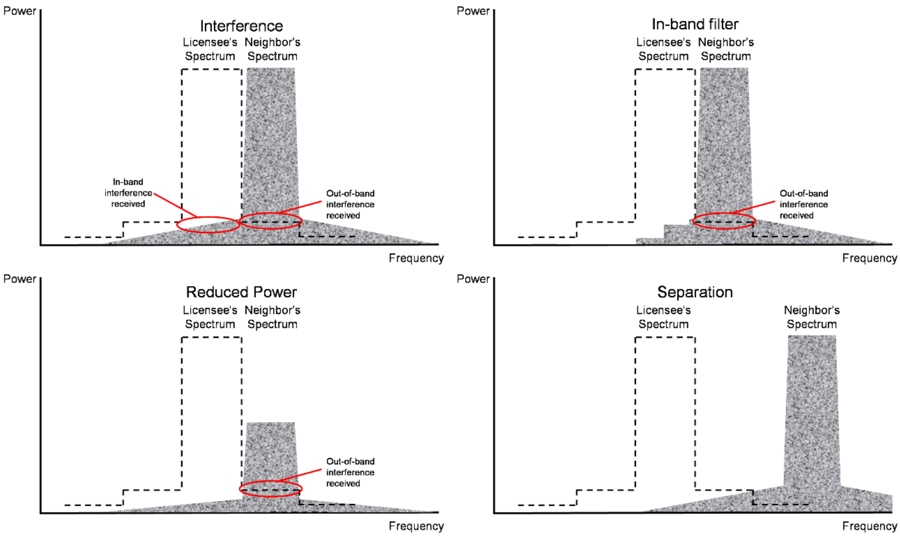
Setting receiver sensitivity standards

When we think of radio spectrum policy, all too often we fail to decompose radio operations into its two fundamental components: transmission and reception. Most spectrum policies regulate transmission thereby controlling unintended reception-interference. No radio device can be perfectly engineered to reject all unwanted signals. Nor can every radio be tuned perfectly to operate on a specific frequency. Radios will emit and receive signals in the band adjacent to it in the spectrum range. Unwanted signals can be classified as coming from two sources: in-band and out-of-

balance between the two, non-mutually exclusive approaches (CARTER, 2007). OFCOM has tried at least one approach to determining the societal need for licence-exempt spectrum (OFCOM, 2005).

band. In-band signals, as the name suggests, are those radio signals which occur within the intended tuning range of the given radio. The source of in-band interference can come from noise in the spectral environment and other authorized users of the band. In-band signals can also come from the spurious emissions of authorized users in adjacent frequency bands and bands which are harmonic in frequency. Therefore, it may be necessary to form guard bands on either side of the centreline frequency. Further, radio receivers also accept some signals from outside of their intended tuning range.

Figure 2 - In-band and out-of-band interference



Source: WIK-Consult

There are three basic solutions to this problem of in-band and out-of-band inference. First is to increase the receivers' robustness to reject unwanted signals. Second is to reduce the power of the interfering signals from adjacent and harmonic frequencies. Third is to further separate the two interfering uses in the tuning range, such as through the use of guard bands. Increasing receiver robustness involving the use of filtering on reception devices and masks on transmitting devices is a technological solution which increases the cost and complexity of radio systems. This has the effect of imposing cost on one radio user, while the benefits accrue to another. However, in some cases in eliminating the source of the interference might not be warranted when the cost and complexity of obtaining interference protection in the form of a licence outweighs the cost of filtering the interfering signals.

There is a trade-off between the width of spectrum blocks and the technologies which give radio equipment the ability to reject undesired signals as noise. Given these constraints, the spectrum management authority must decide how to balance the width of spectrum blocks and impose costs and benefits in a fair, equitable and transparent manner. These determinations are subject to the errors inherent in other administrative determinations.

A novel technological approach to spectrum management which was postulated in the United States more than a decade ago and dubbed the "Interference Temperature", could have empowered technical flexibility, open access and efficiency. First proposed by the U.S. FCC's *Spectrum Policy Task Force* in 2002, the interference temperature metric would have enabled the quantification of interference on a band-by-band basis, by establishing limits on the noise environment in which receivers would be required to operate. The interference temperature represents a maximum cap on the amount of RF energy that lower priority, underlay users could introduce into the band, to the extent it had not already been reached (FCC, 2002, pp. 27-30). As such, it would provide spectrum certain to primary spectrum users in terms of quantifiable level of harmful interference which they could expect within their bands. At the same time this technological solution could encourage efficient use by provided means for enabling sharing by multiple secondary users.

■ Economics of policy determinations

Inherent error in administrative determinations

In public policy debates, the complex organs of society are oftentimes reduced to the hopelessly simplistic dichotomy: markets and bureaucratic institutions. It is widely observed that regulated markets, while imperfect, are more efficient at allocating society's scarce resources than command economies are. Nonetheless, the preference for market institutions and administrative determinations, or vice versa, is driven by ideological concerns rather than the institution's efficiency at achieving a particular goal.

A more apropos inquiry is what are the relative strengths and weaknesses of the institution in achieving desired outcomes (European Commission, 2007). Regulators, especially those with sector-specific expertise, are able to make rational, well informed decisions. However, they lack the information gathering ability and profit incentives of larger, more diverse, and greater numbers than the commercial participants in a market. Therefore, regulators can err in arriving at a decision which is suboptimal.

Decisions regarding spectrum rules must be transparent and objective, considering all feasible options and using all available information on the costs and benefits of these options (MARCUS *et al.*, 2006, p. 13). It might be possible to have transparent and objective adminstrative determinations, but it is generally regarded that other systems might be preferable. Indeed, one observer considers the administrative process for determining licensing rules to be unsatisfactory (CAVE, 2006, p. 224). These limited resources of the regulator lead to asymmetries of information and economic incentives which distort the process. A multitude of self-interested, private actors seem to fare much better. In addition, the information which the regulator receives through the consultation process is tainted at best (BYKOWSKY, OLSON & SHARKEY, 2008b). This is not to suggest that participants in consultation are being wilfully dishonest. Rather, since there are no penalties for over or under- representations of the participants' true valuation of a certain set of rules, there is a perverse incentive to exaggerate as much as is possible without being fraudulent or viewed as simply not credible. Further compounding the problem is the economic phenomenon of regulatory capture. Regulatory capture is said to occur when a governmental agency acts, or appears to act, in a way which favors narrow, private interests, rather than the public interest and enhancing societal welfare (LAFFONT & TIROLE, 1991).

These effects thwart a set of spectrum rules which represents a true social optimum.

Price-guided determinations

Since markets are better suited to efficiently allocating society's resources than are centrally planned economies, price signal information can illuminate the societal costs of alternative spectrum allocations. Price signal information also overcomes the exaggeration problem inherent in the administrative/consultation process as follows. The presence of prices for certain regulatory outcomes imposes penalties for over and under-representations of true valuations. The penalty for a party which under-represents its true valuation of spectrum access would be to not have access to the spectrum. The penalty for over-representation would be the diminishment of the profits a winning bidder could make from the spectrum, by being committed to a bid which represents more than the value of access.

With regard to spectrum policy, price-guided mechanisms tend to promote economically efficient use in several ways. First, auctions, at least in theory, assign radio spectrum licences to those who value it most. Should a higher value user emerge, secondary markets allow those new, potentially more efficient users to acquire access to the spectrum, by motivating initial licensees to divest themselves of a resource for which they have paid substantial sums. Similarly, administrative incentive pricing policies mimic market forces to impose discipline on users, encouraging efficient use.

Markets can be preferable to pure administrative determinations, but when left to their own devices, they can produce perverse results. Yet, economies function best when price signal information is available to prioritize usage. Despite their ability to efficiently allocate resources, markets are highly inadequate to establish social norms and public policy. Perhaps this is due to the corrupting influence of the profit motive and perverse outcomes of individuals working in their own self-interest. All markets require some form of government intervention in order to make them function effectively.

To date, spectrum management authorities have used auctions to assign spectrum licences to their highest value users, but auctions have not been widely used to guide other administrative determinations for spectrum policy. At least four spectrum management authorities have made steps towards introducing price-guided determinations in spectrum allocation and policy. In 2010, the Dutch telecommunications regulator Agentschap Telecom completed a spectrum auction for licenses in the 2.6 GHz band which had two parts. In the first part, bidders vied for a certain amount of spectrum. In the second round, the bidders competed for specific 5 MHz blocks to determine the pairing of the band. ComReg conducted a two-part auction for spectrum in the 26 GHz which shaped both the allocation to specific applications and the assignment to specific users, completed in 2008. In 2006, Ofcom proposed a two-part auction which would not only identify the recipients of spectrum licences but also the pairing determinations associated with those licences. The German auction for UMTS licences in

2000 did not determine the band plan; however, the plan was set by the RegTP based on the results from the auction.

Mathematical model of the problem

In this section, I present a mathematical model which illustrates how spectrum allocations and policy can be determined by participants in a hypothetical auction. The model employs the Shannon-Hartley theorem as an indifference curve for the possible trade-offs between permissible signal strength and allotted channel widths. Under these assumptions, the auction could be used to determine not only the recipients of spectrum licences, but also some of the characteristics of that licence.

Basic elements of the model

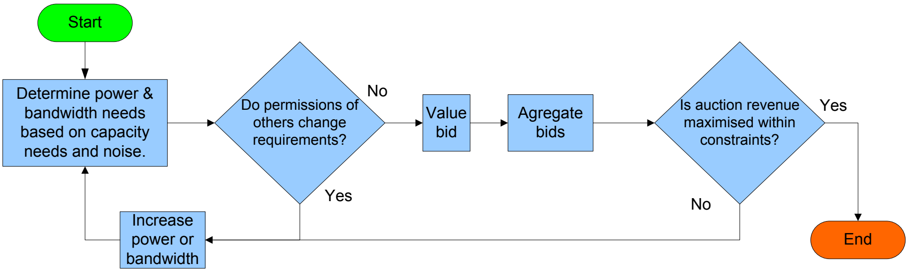
The model is an iterative process and its logic is as follows. The model simultaneously determines the spectrum allocations, policy, and assignments needed by participants in a hypothetical auction. These needs are dictated by what is necessary to satisfy a specified demand for wireless communications ability. The model then values those spectrum allocations, policy and assignments based on the bidders' per unit willingness to pay. Auction revenue in the model is the sum of all the bids for spectrum allocations, policy and assignments. The model is optimized by maximising auction revenue, subject to the constraints of available spectrum and maximum power output. [4] (see figure 3.)

In the model, potential spectrum users participate in a hypothetical auction in which they are free to express their demand for spectrum licences, not just for the licence but also for licences which are different on several dimensions of power, tuning range, and spacing. These considerations are interdependent, affecting one another. For example, the power limits imposed on one user affect the band-edge masking requirements of adjacent users. If a high power use is permitted in one band,

[4] Maximising auction revenue is the objective function only because it determines when demand is satisfied, and no bidders are willing to bid more. The objective of spectrum auctions should not be to raise revenue for the government, but to allocate the spectrum resources efficiently. (See e.g., NOAM, 1998).

the adjacent band will need stricter masking at the edge. Similarly, if channel spacing or channel arrangement is reorganized, this may mitigate the impact of power limits on band-edge requirements.

Figure 3 - Flowchart of basic model



Source: WIK-Consult

In order to explore how the considerations affect one another, I model demand as a function of the ability to send data at a specified transfer rate (bit rate). I use the Shannon-Hartley theorem (explained below) as an indifference curve for the possible trade-offs between permissible signal strength [5] and allotted channel widths. At all points on the curve, bidders are indifferent between having more spectrum and less power, or vice-versa. Further, valuation is a function of noise in the spectral environment. Noise is a function of Gaussian background noise, use in adjacent bands (i.e., adjacent co-channel interference) and shared use of the band. [6]

Thus, the mathematical model shows a hypothetical efficient allocation of several different blocks of spectrum in a frequency range and their assignment. In this way, the auction could determine the organization of the band in question as well as the level of shared or commons use. This might be accomplished by specifying the maximum level of energy permitted in the band (i.e., the "interference temperature") on an underlay or on a sharing basis. The auction could further determine band-edge requirements.

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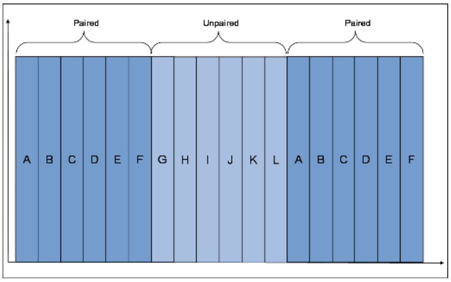
[5] The power dimension, as it is contemplated in the model is receive power; however, it is assumed it to be a proxy for transmit power. Since the model does not simulate any geographic variables, transmit and receive powers are one and the same.

[6] For simplification of the model, we have assumed away harmonic interference.

Figure 4 - Possible outcomes using price-guide determinations



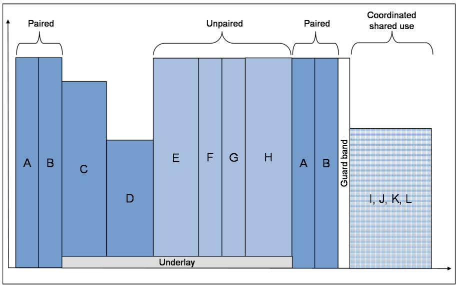


Figure 4 shows two simple examples where price-guided policy could be used to determine allocation, policy and assignment. The image on the left shows how pair and assignment have been determined. Each spectrum user is assigned the same permissible power output (y-axis). This type of result was accomplished in the Agentschap Telecom 2.6 GHz auction, the ComReg 26 GHz Auction and the German UMTS auction. The image on the right shows a somewhat more complicated result. Here, price-guide policy has created a mix of bandwidths permissible power outputs and pairings. In addition, certain assignments will come with the provision that low power licence-exempt use (underlay) is permitted in those bands. Finally, four assignees (I, J, K and L) have been grouped together in a block for shared use. Presumably the parameters of use for this block, such as the coordination protocol and a guard band to protect other assignees, have been determined through price-guided policy.

Shannon-Hartley theorem as indifference curve

I use the Shannon-Hartley theorem (named after Claude Shannon and Ralph Hartley) as the backbone for my mathematical model for two reasons. First, it describes the relationship between the amount, or tuning bandwidth, and the capacity of that channel to carry information, expressed in bits per second. Second, because the theorem relates both signal and noise to channel capacity, I use it to model the effects that independent users have on one another's data rate through the signal-to-noise ratio, and hence on each user's valuation of the spectrum under those conditions.

The Shannon-Hartley theorem quantifies the maximum amount of information that can be transmitted error-free over a communication link (SHANNON, 1949a; SHANNON, 1949b). This channel capacity is a function of: the power level of the signal; the bandwidth of the frequencies employed;

and the presence of noise. The theorem states that channel capacity is a function of bandwidth multiplied by the logarithm of the signal-to-noise ratio. See Formula 1. The signal represents the output of a given radio operating in a given band, measured in watts. Noise is a function of two components: (1) ever-present, non-zero Gaussian noise, and (2) the in-band, adjacent, and harmonic emissions of third-party radios. The Shannon-Hartley theorem is expressed mathematically as:

Formula 1: Shannon-Hartley theorem

$$C = W \log_2\left(1 + \frac{S}{N}\right)$$

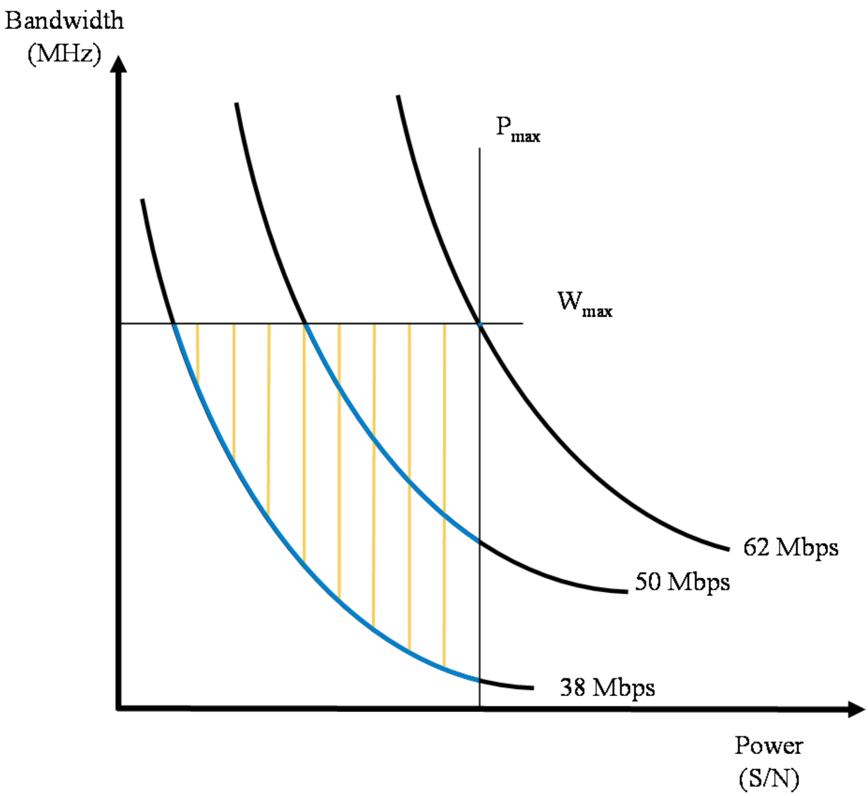
with C = channel capacity in bits per second
W = bandwidth in hertz (cycles per second)
S = signal power watts
N = noise present in watts

For my model, the theorem holds that the capacity to transmit a data file of a given size in a given time across a wireless link can be increased only by either increasing the available bandwidth, or by reducing the signal-to-noise ratio (S/N). The speed of electromagnetic waves which carry the information is fixed depending on the medium through which the waves travel.

Figure 5 shows how the Shannon-Hartley theorem functions as an economic indifference curve in our mathematical model. The figure shows the trade offs between power and bandwidth that produce the same channel capacity.

The y-axis in figure 5 is bandwidth (tuning range) and the x-axis is power (signal-to-noise ratio). The three curves show the trade-offs between power and bandwidth for three spectrum users demanding capacity (data transfer rate) of 38 Mbps, 50 Mbps, and 62 Mbps, respectively. At each point along the curve the spectrum users are indifferent because they can obtain the same channel capacity. At points above their respective curves, the users are better off because they are receiving a higher data rate. However, this comes with the cost of using more spectrum, more power or both. At points below their curves, spectrum users are worse off.

Figure 5 - Shannon-Hartley theorem as indifference curve



Source: WIK-Consult

The line W_{max} represents the maximum allowable spectrum that could be acquired using price-guided policy. [7] The line P_{max} represents the maximum permissible power emissions, for reasons of RF safety. The shaded area shows the possible outcomes using price-guided policy. The blue (light grey in black and white) sections of the curves are possible outcomes for each user, given his/her demand for capacity.

[7] For reasons of market power and competition policy, each individual spectrum user would be limited in the amount of spectrum he/she could acquire.

Modeling the value of exclusive, collective, and licence-exempt use

The model builds on the idea that valuation is directly and positively correlated with tuning range and permissible power levels. Valuations are negatively correlated with noise. The exogenous variables are unit spectrum valuation, noise tolerance, and desired throughput of the wireless link. The endogenous variables include maximum power, tuning width of the blocks, and noise. The constraints are the total bandwidth available for auction, and a cap on the maximum power for reasons of RF safety. The endogenous variables represent the policy determinations which could be determined by the price-guided policy envisioned in this paper.

The model first attempts to calculate the spectrum needs of the hypothetical bidders. There are *k* number of bidders (i.e., *i* = 1, 2, …, *k*). The spectrum needs of each bidder *i* are defined in terms of usable bandwidth (tuning range) and maximum allowable power. These needs are calculated according to Formula 1 as those necessary for a wireless link of certain data transfer rate, C.

Further, the emissions of other users' radios are part of the signal-to-noise ratio, the costs other users' demands impose on those spectrum requirements, and vice-versa. The noise function is specified in Formula 2.

Formula 2: Noise of bidder **i**

$$n_1 = n_o + \frac{S_2}{W_2} \times \frac{1}{2}$$

$$n_i = \left(\frac{S_{i-1}}{W_{i-1}} + \frac{S_{i+1}}{W_{i+1}} \right) \times \frac{1}{2} \quad ; \text{ i = 2, …, (k-1)}$$

$$n_k = \frac{S_{k-1}}{W_{k-1}} \times \frac{1}{2} + n_o$$

with N_i = noise of bidder *i*
W_i = bandwidth of bidder *i*
S_i = power limit of bidder *i*

Once each bidder's spectrum needs are determined, the model calculates the value of the auction revenue from that bidder (his/her bid). Each bidder's valuation is the spectrum it required in terms of bandwidth and power times P – its unit valuation per mega-hertz per watt. This product constitutes each bidder's bid. This is described by Formula 3.

***Formula 3: Auction revenue of bid of bidder* i**

$$U_i = P_i \times W_i \times S_i$$

with U_i = auction revenue of bidder *i*
P_i = unit valuation of bidder *i*
W_i = bandwidth of bidder *i*
S_i = power limit of bidder *i*

For each bidder *i*, noise is calculated as the greater of Gaussian noise present in any communications link or the noise generated by spectrum users in adjacent spectrum blocks. For this 'noise floor', signals other than the ones intended to be received decrease the signal-to-noise ratio. In other terms, the presence of competing signals decreases the communications capacity of the link. This might mean that the bidder i would have use of more power or greater bandwidth. The noise coming from adjacent blocks is based on the permissions allotted to users of those spectrum blocks or adjacent ones.

The auction revenue is the sum of the bids of each bidder *i*. The model would then be optimised to maximise auction revenue. This optimisation function is described in Formula 4 below.

Formula 4: Corresponding optimisation problem

max. $$U = \sum_{i=1}^{k} U_i = \sum_{i=1}^{k} p_i \cdot \left[W_i \cdot S_i\right]$$

s.t. $$\sum_{i=1}^{k} W_i \le W_{\max}$$

$$S_i \le S_{\max} \left(\forall i = 1, \ldots k\right)$$

with U = total auction revenue
U_i = auction revenue of bidder *i*
P_i = unit valuation of bidder *i*
W_i = bandwidth of bidder *i*
S_i = power limit of bidder *i*
k = number of bidders
Model Assumptions:

Variables: P_i , W_i (decision of each bidder *i*)
Exogenous parameters: W_{max}, S_{max}, C_i, n_o
Model results: n_i, S_i, U_i, U
with C_i = data rate of bidder *i* (i = 1, …, *k*)
n_i = noise of bidder *i*
n_o = background noise

Formula 5: Transformation of the optimisation problem

$$\text{max.} \quad \sum_{i=1}^{k} P_i W_i n_i \left(2^{\frac{C_i}{W_i}} - 1 \right)$$

$$\text{s.t.} \quad \sum_{i=1}^{k} W_i \leq W_{\max}$$

$$\sum_{i=1}^{k} n_i \left(2^{\frac{C_i}{W_i}} - 1 \right) \leq S_{\max}$$

Valuation of a single spectrum block is determined by the uses in the adjacent blocks, which are in turn influenced by the use in the first block. The optimization process ensures that spectrum allocations and policy are awarded to the highest value users.

Solving the optimization of the model is incredibly complex. Any such an optimization requires examining thousands of possible outcomes in search of the allocation, policy and assignment which maximises auction revenue. This does not suggest that such an auction is impossible. Indeed, the process is not harder in principle than the current administrative process. Each of the optimizations necessary to complete the model would still be present in an administrative proceeding, and the spectrum management authority would have to evaluate each of them without the benefit of mathematical guidance or price signals (POGOREL, 2007).

A proper optimization might employ computer-based simulation with independent bidding agents. Alternatively, a solution could be achieved using experimental economics. Experimental economics is an emerging discipline which seeks to study economic behavior by creating an "economic environment" and asking live subjects to make decisions to simulate payoffs. Using experimental economics, future research might assign the valuations we have created for each of the bidders to "game players" and allow them to participate in a simulated economic environment.

An earlier version of this article presented a simplified MS Excel-based implementation of the model to demonstrate the validity of the model as a proof of concept (CARTER, 2009). The auction format of the proof of concept model was that of a first-price, simultaneous, multi-round, ascending auction. The model linked the usage and valuation of 10 hypothetical bidders for a particular 100 MHz-wide band to be auctioned. Variables for bidders' valuation in the model were parameterized by using the results of the 2000 German UMTS auction. The optimization to maximize auction revenue was subject to the following three constrains: (1) the maximum power afforded to any one bidder was 100 Watts; (2) total bandwidth awarded to all bidders could be no more than 100 MHz; and (3) bandwidth was bid for in minimum increments of 1.25 MHz [8]. Total auction revenue in the model was only 13% of the €50.80 billion in the actual 2000 auction. A run of the model established a baseline result. In all, 51.25 of 100 MHz were assigned, resulting in total auction revenue of €7.81 billion. Each bidder received at least some assignment of spectrum (between 2.5 and 10 MHz each). There was also a mix of high and low power users and some spectrum available was not assigned to bidders because the individual assignments more closely matched individual needs than would have been the case had the allocations been completed by administrative process. This left over spectrum represents an efficiency gain. [9] Further, at the conclusion of the auction, additional spectrum resources are available for assignment to public sector or commons uses.

Conclusion

The mathematical model presented in this article illuminates one of several possible implementations of an auction that could be used in place of the administrative determinations necessary to ensure efficient spectrum allocation and policy, in addition to assignment of spectrum licenses. As compared to conventional spectrum auctions, price-guided policy for determining allocations and policy would arrive at an assignment of spectrum rights to the highest value users as well as ensure that the contours of those rights are more efficient than those which could be

[8] The 1.25 MHz minimum requirement has the necessary effect of reducing the total bandwidth demanded, since bidders might purchase slightly more spectrum, if they could do so in smaller increments.

[9] Indeed, the ComReg auction eft 4 lots of 2 x 28 MHz spectrum unassigned after the auction.

achieved by using market mechanisms solely for assignment. As such, this price-signal information can be used to mitigate error in administrative determinations. It also helps to ensure technological and service neutrality. However, a price-guided approach to allocations and policy would not be possible in bands subject to international harmonizations. In bands which are not harmonized, the most actionable initial implementations include: band planning, block bandwidth, duration of rights and maximum power output determinations. Other implementations could accomplish the determinations of: band-edge requirements; guard bands; exclusivity of use; underlay characteristics such as the maximum interference temperature; and possibly congestion-based protocols.

Price-guided policy could be used to establish a market-clearing price for noise tolerance, e.g. the cost of interference. Since noise is influenced by the presence of competing uses, the level of noise tolerance represents a spectrum user's preference for having either exclusive or shared use of the spectrum. In terms of technology, the noise tolerance is equivalent to the sensitivity to installing reception masks for adjacent co-channel interference and the presence of other low power users in the band (either as an underlay or co-primary users).

Price-guided policy encourages an efficient outcome for several reasons. First, price-guided policy would improve allocative efficiency of limited spectrum resources. Second, price-guided policy mitigates the allocative errors inherent in administrative determinations. Bidders can acquire exactly the set of spectrum rights they need, instead of a set determined by an administrative decision. These differentiated spectrum inputs could lead to differentiated networks and services in the market for wireless communications services. This in turn would lead to lower prices and networks which more closely match heterogeneous user demands. Further, in instances where the cost of coordinating interfering uses with other spectrum users is low, price-guided policy would allow users to acquire spectrum on a non-exclusive basis. This would enable certain users to share the cost of a spectrum licence, reducing the up-front cost of obtaining access to the band. Finally, the auction is not incompatible with spectrum trading in the future. Price-guided determinations are a viable means for initial assignment of potentially tradable rights. This would not preclude the possibility of the auction being two-sided, whereby existing licensees could tender their licences as part of a massive band reorganization.

The model presented here examines price-guide policy as a means for initial assignment of tradable rights. The tradability of these is possible so long as the transferor does not convey more than the rights (along all dimensions such as bandwidth, power, exclusivity, duration of rights and other parameters) it has acquired. Such price-guided policy could also be used to determine efficient allocation, policy and assignment of spectrum in conjunction with a two-sided combinatorial auction such as the one proposed by the U.S. FCC researchers Williams and Kwerel (KWEREL & WILLIAMS, 2002). The Williams and Kwerel auction, sometimes referred to as the "Big Bang" auction or "incentive auctions", is intended to reallocate spectrum to flexible use by organising a large-scale, two-sided auction in which existing licensees voluntarily offer already assigned spectrum licences to be auctioned together with presently unassigned spectrum. The Willams and Kwerel approach is now being implemented by the FCC in its incentive auctions proceeding to reallocate the broadcast TV spectrum to mobile broadband uses (FCC, 2012). Because the auction would make complementary spectrum bands available in a single auction, it could reallocate and restructure those bands efficiently.

Given the complexities of auction design and strategic behavior by auction participants, prior to an initial implementation of any price-guided policy, further research must be completed as to what the appropriate auction format is. However, price-guided policy such as the model described here holds substantial promise for creating efficient allocations and assignment of spectrum.

References

BYKOWSKY, M., CARTER, K. R., OLSON, M. & SHARKEY, W. (2008): "Enhancing Spectrum's Value via Market-Informed Congestion Etiquettes", FCC OSP Working Paper Series. http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-280521A1.pdf

BYKOWSKY, M., OLSON, M. & SHARKEY, W.:

- (2008a): "Modeling the Efficiency of Spectrum Designated to License Use and Unlicensed Operations", FCC OSP Working Paper Series. http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-280522A1.pdf
- (2008b): "A Market-Based Approach to Establishing Licensing Rules: Licensed versus Unlicensed Use of Spectrum", FCC OSP Working Paper Series. http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-280523A1.pdf

CARTER, K. R.:

- (2007): "Policy Lessons from Personal Communications Services: Licensed vs. Unlicensed Spectrum Access", *CommLaw Conspectus* Vol. 15, Issue 93.
- (2009): "Next Generation Spectrum Regulation for Europe: Price-Guided Radio Policy", WIK Diskussionsbeitag, November.

CARTER, K R., LAHJOUJI, A. & McNEIL, N. (2003): "Unlicensed and Unshackled: A Joint OSP-OET White Paper on Unlicensed Devices and Their Regulatory Issues", FCC OSP Working Paper Series.
http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-234741A1.pdf

CAVE, M. (2006): "New spectrum-using technologies and the future of spectrum management: a European policy perspective," in *Communications: The Next Decade*, Richards, E., Foster, R. & Kiedrowski, T. (Eds), Ofcom.

COASE, R. H. (1959): "The Federal Communications Commission", 2 J.L. and ECON. 1.

Commission for Communications Regulation:
- (2005): "Response to Consultation, 26 GHz Fixed Service Band – Spectrum", Document No: 05/84.
- (2008): "Information Memorandum, The Award of National Block Point to Point and Point to Multipoint Assignments in the 26 GHz Band", Document No: 08/93R.

European Commission:
- (2005): "Communication from the Commission to the Council, the European Parliament and the European Economic and Social Committee and the Committee of the Regions, A market-based approach to spectrum management in the European Union", 14 COM(2005) 400 final.
- (2007): "Proposal for a Directive of the European Parliament and of the Council amending Directives 2002/21/EC on a common regulatory framework for electronic communications networks and services, 2002/19/EC on access to, and interconnection of, electronic communications networks and services, and 2002/20/EC on the authorisation of electronic communications networks and services", COM(2007) 697 final.

FCC - Federal Communications Commission:
- (2012): "In the Matter of Expanding the Economic and Innovation Opportunities of Spectrum Through Incentive Auctions", Notice of Proposed Rulemaking, Docket No. 12-268. http://www.fcc.gov/incentiveauctions
- (2007): "Memorandum Opinion and Order, In the Matter of Wireless Operations in the 3650-3700 MHz Band Rules for Wireless Broadband Services in the 3650-3700 MHz Band Additional Spectrum for Unlicensed Devices Below 900 MHz and in the 3 GHz Band", ET Docket No. 04-151, WT Docket No. 05-96, ET Docket No. 02-380. http://hraunfoss.fcc.gov/edocs_public/attachmatch/FCC-07-99A1.pdf
- (2002): *Spectrum Policy Task Force Report*.
http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-228542A1.pdf

GRIMM, V., RIEDEL, F. & WOLFSTETTER, E. (2001): "The Third Generation (UMTS) Spectrum Auction in Germany", CESifo Working Paper No. 584.

HARDIN, G. (1968): "The Tragedy of the Commons", 162 SCI. 1243.

JEHIEL, P. & MOLDOVANU, B. (2001): "The European UMTS/IMT-2000 License Auctions", University of Mannheim Working Paper.

KWEREL, E. & WILLIAMS, J. (2002): "A Proposal for a Rapid Transition to Market Allocation of Spectrum", FCC Office of Plans and Policy Working Paper Series. http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-228552A1.pdf

KOMESAR, N. K. (1997): *Imperfect Alternatives: Choosing Institutions in Law, Economics, and Public Policy*, University of Chicago Press.

LAFFONT, J. J. & TIROLE, J. (1991): "The politics of government decision making. A theory of regulatory capture", *Quarterly Journal of Economics* 106(4): 1089-112.

MARCUS, J. S., NETT, L., SCANLAN, M., STUMPF, U., CAVE, M. & POGOREL, G. (2005): *Towards More Flexible Spectrum Regulation*, a WIK study for the German BnetzA. http://www.bundesnetzagentur.de/media/archive/4745.pdf

MARCUS, J. S., BURNS, J., HANSEN, P., MARCUS, M., MARKS, P., PUJOL, F. & REDMAN, M. (2006): "Study on Legal, Economic, & Technical Aspects of 'Collective Use' of Spectrum in the European Community", a study on behalf of the European Commission.
http://europa.eu.int/information_society/policy/radio_spectrum/docs/workshop_collective_use/cus_rep_fin.pdf

NETT, L.:
- (2000): "UMTS-Lizenzvergabe in Deutschland: Eine spektakuläre Auktion in der Wirtschaftsgeschichte", in *WIK Newsletter*, pp. 18-19.
- (2001): "Marktorientierte Allokationsverfahren für Nummern", WIK Discussion Paper No. 213.

NOAM, E.:
- (1995): "Taking the Next Step Beyond Spectrum Auctions: Open Spectrum Access", *IEEE Communications Magazine*.
- (1998): Spectrum Auctions: Yesterdays Heresy, Today's Orthodoxy, Tomorrow's Anachronism", *Journal of Law and Economics*, pp. 765-790.

OFCOM (2005): "Spectrum Framework Review".

POGOREL, G. (2007): "Nine Regimes of Radio Spectrum Management: A 4-Step Decision Guide", *Communications & Strategies*, no. 65, 1st Q. 2007, pp. 169-183.

RegTP (2000): "Entscheidung der Präsidentenkammer vom 18.02.2000 über die Regeln für die Durchführung des Versteigerungsverfahrens zur Vergabe von Lizenzen für UMTS/IMT- 2000", Mobilkommunikation der dritten Generation-Aktenzeichen: BK-1b-98/005-2, Bonn.
http://www.bundesnetzagentur.de/media/archive/352.pdf

SHANNON, C. E.:
- (1949a): *The Mathematical Theory of Communication*
- (1949b): "Communication in the presence of noise", *Proc. Institute of Radio Engineers*, vol. 37 (1): 10-21.

SNIDER, J. H. (2006): "Spectrum policy wonderland: a critique of conventional property rights and commons theory in a world of low power wireless devices", Telecommunications Policy Research Conference, Arlington, VA.
http://web.si.umich.edu/tprc/papers/2006/644/06-09-30--TPRC--SpectrumPolicyWonderland.pdf

SOKOL, D. D. (2001): "The European Mobile 3G UMTS Process: Lessons From the Spectrum Auctions and Beauty Contests", *Virginia Journal of Law and Technology*, Volume 6, Issue 17.